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Seeking carbon-consistency in the climate-science-to-policy interface

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Abstract The multi-faceted nature of anthropogenic climate change means that policy analysis must draw on a very wide range of disciplines. In combining information from a variety of sources, it is important to ensure consistency across the interfaces. This paper draws on some historical cases from carbon cycle studies in order to illustrate the type of issue involved. These are used as background for presenting current issues that arise in analysing changes in atmospheric carbon dioxide and the specification of emissions that cause these changes. Implications for consistent analysis of consequent climate changes are noted.

Keywords Carbon cycle · Modelling · Policy consistency

Context

Analyses of anthropogenic global change need to combine information from disparate sources and disciplines. This imposes severe demands on achieving consistency in specifying the interfaces between the various concepts and data streams that flow into modelling and analysis. Even within the restricted topic of modelling the changing global carbon cycle, issues of inconsistency have caused problems in the past, and care is required to avoid such problems in the future.

The causes of uncertainty in empirical quantities that underly various types of modelling have been characterised (Morgan and Henrion 1990, Sect. 4.4) as:

- statistical variation—particularly from measurement error;
- subjective judgment—e.g. on validity of data;
- linguistic imprecision—e.g. in relating qualitative features onto quantitative measures;
- variability—in space and/or time;
- inherent randomness of the processes involved;
- approximation—in both model building and model analysis;
- disagreement—particularly over the relative importance of other forms of uncertainty.

Since much of our information about natural systems is obtained indirectly [involving various forms of 'inverse problem' (Tarantola 1987)] there is extensive scope for compounding these various forms of uncertainty, given the 'error amplification' that is associated with many inverse problems. This makes 'consistency' in analyses of the carbon cycle particularly important.

The outline of this paper is as follows. Following Enting (1987), 'Representing the carbon cycle' section describes the disparate forms of carbon cycle modelling in terms of a modelling spectrum. This provides a unifying framework within which to consider issues of

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consistent treatment of the carbon cycle. 'Historical examples' section presents earlier examples where consistency issues have proved pivotal to the resolution of anomalies: carbon budgeting clarification (Sarmiento and Sundquist 1992), consistent treatment of deforestation fluxes (Enting et al. 1994), and consistency in ^{14}C budgeting (Lassey et al. 1996). A range of issues associated with the consistent specification of CO_2 emissions are analysed in 'Consistency in emission specifications' section. Current issues that are explored in 'Current issues in global carbon cycle research' section involve the consistent propagation of uncertainty from past to future, and the role of climate-to-carbon feedbacks (Friedlingstein et al. 2006). 'Concluding remarks' section note how many of the issues addressed here in connection with describing the global carbon cycle have analogues in highly resolved regional modelling of the carbon cycle using multiple data streams and in other aspects of modelling global change.

As well as illustrating issues with consistent definitions, the various examples illustrate issues of consistent specification of uncertainties. The concluding remarks note how both these aspects have importance in studies of global change far broader than the carbon cycle.

Representing the carbon cycle

The carbon cycle involves a multiplicity of processes and interacts with human systems in diverse ways (Field and Raupach 2004). Many disparate forms of modelling have been used to represent the behaviour of the carbon cycle and its interactions with human systems. One way of characterising the diversity of representations of the carbon cycle is in terms of a spectrum of modelling (Karplus 1977) running from 'black-box' models to 'white-box' models representing a range from empirical/stochastic to deterministic/mechanistic models. Enting (1987) used this concept to describe carbon cycle models as spanning the range:

- Curve-fitting of CO_2 trends, ignoring any relation to emissions.
- Assuming that the airborne fraction (the proportion of CO_2 emissions that remains in the atmosphere) is constant, approximating the relation between emissions and concentrations.
- Using response functions (see Eq. 1 below), to capture the time-dependence of the functional dependence on emissions. (i.e. valid over a wider range of future conditions than assuming a constant airborne fraction and indicating that a constant airborne fraction applies for exponential growth in emissions).
- Lumped 'Box models', linking processes and diagnostic quantities such as the carbon isotopes: ^{14}C and to a lesser extent ^{13}C.
- Disaggregating boxes to resolve major biomes and major ocean water masses.
- Carbon resolved at full climate model resolution and driven (where relevant) by climate model processes.

However, even mechanistic models will often be found to have empirical submodels.

The response function form, introduced into carbon cycle modelling by Oeschger and Heimann (1983), provides a link between statistical and deterministic modelling. The atmospheric CO_2 concentration, $C(t)$, at time t is written:

$$C(t) = C(t_0) + 0.47 \int_{t_0}^{t} R(t - t')S(t')dt' \qquad (1)$$

where t_0 is the end of a notional pre-industrial equilibrium, $S(t)$ is the anthropogenic emissions and $R(t)$ is a response function that gives the amount of CO_2 remaining in the atmosphere at time t after a unit emission. The factor of 0.47 represents a conversion to ppm concentrations for $C(t)$ from Gt carbon per unit time, for $S(t)$, making $R(t)$ dimensionless. (In the original study by Oeschger and Heimann, $R(t)$ represented only the effect of the oceans, so that $S(t)$ also included net terrestrial fluxes.)

Response functions have provided the basis of a statistical deconvolution through the Kalman Filter formalism (Trudinger et al. 2002), representing $R(t)$ as a sum of exponentials. This allows the response to be represented as a sum of terms with first-order decay (Wigley 1991) and thus readily represented as a linear state-space model for the Kalman Filter formalism. The actual study by Trudinger et al. applied the more general Extended Kalman Filter, as required since they used a model with weak non-linearity in CO_2 exchanges.

The linear form of (1) has also provided a basis for the analysis of information content in carbon data using a Laplace transform representation (Enting 2007), indicating the relatively small amount of information in the CO_2 concentration record. The utility of the Laplace transform in this context comes from the transform's property of mapping integro-differential relations into algebraic relations and the particular history of quasi-exponential growth in CO_2 emissions. The Laplace transform of the response is defined by

$$r(p) = \int_0^\infty \exp(-pt)R(t)dt \qquad (2)$$

with corresponding relations giving $s(p)$, the Laplace transform of $S(t)$, and $q(p)$ which is the Laplace transform of a carbon mass perturbation, $Q(t) = [C(t) - C(t_0)]/0.47$. Laplace transforms of convolutions are given by products of Laplace transforms, so that (1) implies

$$q(p) = r(p)s(p) \qquad (3)$$

In these terms the airborne fraction is given by $p\, r(p)$ (Enting 1990, 2007). This airborne fraction applies when emissions grow as $\propto \exp(pt)$, i.e. with growth rate p. Since emissions had approximately exponential growth over the twentieth century, the relation between $C(t)$ and $S(t)$ primarily constrains the response, $R(t)$, by specifying its Laplace transform, $r(p)$, at one particular growth rate. The concentration data, $C(t)$, give little information about the behaviour of the carbon cycle on other time scales (Enting 2007). Indeed the main features of carbon exchanges had been identified on the basis of ^{14}C data (Broecker et al. 1980, for example) at a time when the directly measured CO_2 record (from 1958 onwards) was relatively short, and CO_2 data from ice cores were not yet available. Lumped box models have been the main tool for inferring carbon cycle properties from isotopic data.

Historical examples

The history of carbon cycle studies provides several examples that can illustrate the importance of consistent specifications.

Around 1990, low estimates of oceanic uptake of carbon were obtained both from ^{13}C (Tans et al. 1993) and more directly from p_{CO2} combined with inversions (Tans et al. 1990). In contrast, high oceanic carbon uptake was estimated both from ocean mixing models and from ^{13}C inventories (Quay et al. 1992). The discrepancy was resolved by the realisation that: (i) the high estimates represented the net rate of ocean carbon accumulation, while (ii) the low estimates represented the net air-sea carbon flux, and (iii) that these quantities would differ because of the natural carbon flux into the oceans via rivers (Sarmiento and Sundquist 1992). A consistent CO_2–$^{13}CO_2$ budget was presented by Heimann and Maier-Reimer (1996). Apart from identifying conceptual issues of defining the carbon budget, Sarmiento and Sundquist (1992) provided corrections to the study by Tans et al. (1990). They noted the role of a surface temperature effect (Robertson and Watson 1992) in the determination of fluxes from p_{CO2} and a correction for CO transport (Enting and Mansbridge 1991) in the inversion calculations. For a summary, and a broader perspective on the history of carbon cycle modelling, see (Enting 2002, Sect. 14.1). Enting (2000) has discussed the need for statistical consistency when combining multiple data sets for interpreting carbon cycle behaviour.

However, even given a consistent carbon budget, there is a further requirement for consistency in how the budget is balanced in terms of the net terrestrial carbon sink (Wigley 1993). The modelling issue is whether this sink reflects:

– random influences (e.g. from climate fluctuations) for which the best future estimate is zero; or
– a systematic influence (such as CO_2-enhanced growth) that can be expected to continue into the future.

Of course these are two extreme positions, with reality most likely to represent some intermediate case. CO_2 modelling reported in the first IPCC assessment was generally based on the first case, while modelling for the second assessment generally used the second case. Wigley referred to the two cases as 'no-feedback' and 'feedback' cases, regarding CO_2-enhanced growth as an atmosphere-to-biosphere feedback. This was notionally distinct from the climate-to-carbon feedbacks. However, as discussed below, fitting twentieth century CO_2 data may well imply that a model fit of CO_2-enhanced growth is in part acting as a proxy for climate-to-carbon feedback effects. Early discussions by Wigley also used the terms 'unbalanced' and 'balanced' for the two forms of budget, but a more appropriate terminology is to regard the difference as a question of how the budget is balanced (T.M.L. Wigley, personal communication).

A further example of the importance of consistency conditions comes from calculations for the IPCC Radiative Forcing report (Schimel et al. 1995), where a combined modelling study noted that a correction term was needed to deal with estimated emissions from changes in land-use. These were net emissions and included a 'regrowth' component (Houghton et al. 1983; Houghton 1991). However, some models already included regrowth processes. To remove the inconsistency from such 'double-counting', at least to first-order, a correction term was derived to convert net fluxes to gross fluxes (Enting et al. 1994).

Some of these issues recur when considering consistent treatment of ^{14}C in the calibration of carbon cycle models (Lassey et al. 1996). The analysis by Lassey et al. found a consistent CO_2–$^{14}CO_2$ budget, in contradiction to earlier studies by Hesshaimer et al. (1994). Some of the consistency issues identified by Lassey et al. were those of initial conditions and biospheric exchange. A lesser reliance on early stratospheric ^{14}C data (on the basis of sampling considerations) also contributed to the ability to find a consistent budget.

Consistency in emission specifications

Two of the examples above can be regarded as examples of more general questions about consistency in emissions of CO_2 (and ^{14}C):

- How can emission scenarios for the future be specified in a way that is consistent with an uncertain past?
- What does the modeller regard as internal response versus external forcing? The example above, concerning the land-use flux, hinged on how regrowth after clearing was classified. Similar questions arise with the classification of feedbacks—describing a system output as feeding back into the input implies a definition of what is 'inside' the system and what is 'outside'.

There is no universal 'right' answer to these questions—what is essential is that any chosen answer be applied consistently.

The convolution defined by Eq. 1 can be split at a time t_1 representing the 'present' (or the end of available data) as:

$$[C(t) - C(t_0)]/0.47 = \int_{t_0}^{t_1} R(t-t')S_1(t')dt' + \int_{t_1}^{t} R(t-t')S_2(t')dt' \qquad (4)$$

Given that $C(t)$ is unknown for $t > t_1$, the function $S_2(t)$ from scenario contains no information about $S_1(t)$ and so specification of S_2 does not reduce the range of uncertainty in S_1. One way that this could be addressed is to note that results from using S_2 are not only conditional on the use of S_2 (as in any use of scenarios) but also conditional on a particular estimate of S_1. A more sophisticated approach would be to define scenarios in terms of projected rates relative to the past, so that a range of estimates of past emissions implies a range of future emissions.

However, a more typical practice is to largely ignore the consistency issue and accept the discontinuity, except that some degree of ad hoc interpolation might be applied for a decade or two into the future.

Some other issues involving the specification of future emissions (either prescribed through scenarios or deduced from concentration targets) in a manner consistent with the past are:

- Distinguishing natural fluxes from anthropogenic emissions, especially when non-linear processes are involved;
- Achieving a consistent synthesis of emission profiles derived from concentration targets versus achievable cases based on scenarios;
- Reconciling disparate inventories of past emissions in a way that gives a consistent representation of pointwise uncertainties and their correlations over time;
- Using ad hoc modelling relations between CO_2 and non-CO_2 gases and/or aerosols (indirect greenhouse gases) (Wigley et al. 1996);

The problem of defining multi-gas targets for mitigation of climate change is more general than the carbon cycle studies describe here, but a number of the difficulties that arise are due to specific aspects of the behaviour of the carbon cycle. In terms of $R(t)$, the response function in Eq. 1: (a) $R(t)$ is not a simple exponential decay, i.e. CO_2 is not characterised in terms of an atmospheric lifetime; (b) in particular, $R(t)$ does not go to zero, except possibly on geological time scales (Sundquist 1985). Numbers given for the atmospheric lifetime of CO_2 should be regarded as 'summary statistics' that are context-dependent and unsuitable for computation.

Analysis of trade-offs in mitigation of the different greenhouse gases requires some definition of 'equivalence'. However, 'equivalence' depends on the context. Two specific forms of equivalence, both referenced to CO_2, have been important: 'emission equivalence' and 'concentration equivalence'. Concentration equivalence is the simpler of the two. A combination of gases, including CO_2, is assigned a CO_2-equivalent concentration given by the amount of CO_2 that, on its own, would have the same radiative forcing as the combination of gases. Thus, as indicated by the IPCC definition (Houghton et al. 2001, glossary), CO_2 concentration equivalence *is a property of the mixture of gases*.

CO_2-equivalent emissions are defined in terms of their cumulative radiative forcing, integrated over a specified time horizon and taking into account natural loss processes. This quantity is termed the absolute global warming potential (AGWP). The global warming potential (GWP) is defined by scaling the AGWP so that CO_2 has a GWP of 1. A time horizon of 100 years is commonly used for GWPs and provides the basis for the definition of CO_2-equivalent emissions used in the Kyoto Protocol. The differences between the different forms of CO_2-equivalence mean that the 'Kyoto' definition (emission equivalence using 100-year GWPs) will not be appropriate for defining targets for stabilising human impact on climate, in terms of carbon offsets based on changes in other gases.

A major difficulty in defining appropriate mitigation targets is that the costs of mitigation are defined mainly in terms of emissions, $S(t)$, while the costs of adapting to the consequences of insufficient mitigation are more closely related to future temperatures, $T(t)$, with the functional relation between emissions and temperatures still subject to significant uncertainty.

Of course, there also needs to be consistency in discussing targets, distinguishing between:

– a global emissions target;
– a target for developed nations as a group;
– a target for a nation with above-average per capita emissions.

However, this is more of an issue of politics and communications rather than a 'science consistency' issue.

Current issues in global carbon cycle research

Two issues that are particularly important in understanding the role of the carbon cycle in global climate change over the twenty-first century are:

– consistent propagation of uncertainties between past and future;
– the role of climate-to-carbon feedback.

Of the various aspects of uncertainty listed by Morgan and Henrion (1990), the issue of 'linguistic precision' (possibly reflecting 'subjective judgment') most closely captures the concerns of consistency that are the subject of the present paper. Other aspects of uncertainty are more closely related to uncertainties in the models that are used to encapsulate our knowledge of the carbon cycle.

While Evans and Stark (2002) have noted the appropriateness of non-parametric statistical techniques in analysing inverse problems, model calibration is most frequently undertaken in a parametric form with model uncertainty captured in terms of parameter uncertainty. The concept of the modelling spectrum can be used to explore the limitations of parametric approaches by analysing how the information use in calibrating parametric lumped models corresponds to information use in non-parametric statistical approaches.

A generic approach can be applied to a model whose vector of parameters, $\mathbf{x}$, is calibrated by minimising some objective function $\Theta(\mathbf{x})$. The range of uncertainty of a dependent quantity $f(\mathbf{x})$ (e.g. future CO_2) can be explored by using a Lagrange multiplier, Λ, and exploring the minima of $\Theta^*(\mathbf{x}) = \Theta(\mathbf{x}) - \Lambda f(\mathbf{x})$ for range of Λ. Thus for each Λ, parameter values $\mathbf{x}(\Lambda)$ lead to $f(\mathbf{x}(\Lambda))$, with the range of Λ being determined by the 'goodness-of-fit' as measured by $\Theta(\mathbf{X}(\Lambda))$. This approach was used in global carbon cycle studies by Enting and Pearman (1987).

The generality of the Lagrange multiplier formalism comes at a significant computational cost. Simplifications may be possible in special cases. In particular, Enting and Lassey (1993) noted that with the available data, carbon cycle uncertainty had a dominant direction of uncertainty, associated with the terrestrial component of the carbon budget. This reflected the uncertain sizes of the sink from terrestrial uptake (the so-called CO_2-fertilisation) and the source from land-use change. In contrast, given clarification of the definitions, oceanic uptake was relatively well-determined from the oceanic uptake of ^{14}C and the use of ^{13}C (and in later studies, changes in atmospheric O_2:N_2 ratios) to determine how net CO_2 uptake was partitioned between land and ocean.

The relations can be illustrated by partitioning the Laplace transform equation, using (3) to define sources. These are expressed as a sum of 'Fossil' and 'LUCF' (land-use change and forestry) components. The combined oceanic and terrestrial response to CO_2 sources can be written as a combination of oceanic and terrestrial responses (Enting et al. 1994; Enting 2007). Thus:

$$\begin{aligned} s(p) &= s_{\mathrm{Fossil}}(p) + s_{\mathrm{LUCF}}(p) = q(p)/r(p) \\ &= q(p)[1/r_{\mathrm{Bio}}(p) + 1/r_{\mathrm{Ocean}}(p) - p] \end{aligned} \quad (5)$$

With ocean response constrained by ^{14}C, uncertainties in $s_{\mathrm{LUCF}}(p)$ map directly onto uncertainties in $r_{\mathrm{Bio}}(p)$, the response of the terrestrial biosphere. A similar strong relation between emissions estimates and the estimates of enhanced photosynthesis was found in a study by Ricciuto et al. (2008) using a more sophisticated calibration technique (Markov Chain Monte Carlo) but a restricted data set (with no direct use of isotopic data).

The form of the Enting and Lassey model, and the reduced form used in later studies (Enting and Trudinger 2002), had net primary production (NPP) as a function of CO_2 concentration with respiration represented by a linear response, $r_{\mathrm{Resp}}(p)$. Linearising the NPP perturbation as $\beta Q(t)$ leads to

$$r_{\mathrm{Bio}}(p|\beta) = 1/[p(1 + \beta r_{\mathrm{Resp}}(p))] \quad (6)$$

where the conditional notation notes the parametric dependence explicitly. The range of responses given by a range of β has been plotted in terms of biosphere:atmosphere partition factors in Fig. 1 of Enting (2007).

Including the β-dependence in the relation (3) gives two distinct ways of representing future changes.

The first way, represented by

$$q(p|\beta) = [s_1(p|\beta) + s_2(p)]r(p|\beta) \quad (7)$$

has past emissions, s_1, whose uncertainties imply uncertainties in the response (represented by the parametric dependence in β in each case). The future emissions, s_2 are taken as exact, and so the uncertainties in the response imply uncertainties in concentrations, as denoted by the dependence $q(p|\beta)$.

The second form applies when determining emissions required to achieve concentration targets. It is represented by

$$q(p) = [s_1(p|\beta) + s_2(p|\beta)]r(p|\beta) \quad (8)$$

The concentrations, past and future, are regarded as precisely specified. The uncertainties in past emissions s_1 imply uncertainties in response r and this in turn implies uncertainties in future emissions, s_2. In this type of calculation it is convenient to represent the concentration curves as smoothing splines (Enting et al. 2008). These have continuous second derivatives in the concentration which implies continuous first derivatives in emissions, i.e. no abrupt changes in emission growth rate.

An example of the distinction occurs in a recent study of the climate benefit from geosequestration (Enting et al. 2008) which involved three different treatments of emissions. Conditional on the temperature model, the cases were:

- **IS92a:** emissions exact, concentrations (and temperature) reflect budget uncertainty;
- **stabilisation at 500 ppm:** concentrations (and temperature) exact, emission uncertainty reflects budget uncertainty;
- **perturbation from geosequestration:** emission perturbation exact, perturbation to concentration (and temperature) reflects budget uncertainty.

The IPCC 4th Assessment Report noted that, along with uncertainties in ice-sheet processes, the effect of climate-to-carbon feedbacks represents one of the largest uncertainties in twenty-first century climate change (Alley et al. 2007; Denman et al. 2007). In considering feedbacks where global warming affects the global carbon budget, it is important to appreciate that such feedbacks will have occurred over the twentieth century, since CO_2 increases have been accompanied by warming. Therefore to the extent that carbon models have been calibrated to twentieth century behaviour, other processes will have acted as proxies for feedback effects. Results for CO_2 projections for 2100 (given emission scenario IS92a), as reported by the IPCC (Houghton et al. 2001, Appendix 2) for the Bern-CC model (Joos et al. 2001) span the range 632 to 902 ppm for the 'low' and 'high' feedback cases. For comparison, using the IS92a emissions the earlier Bern model (without explicit climate-to-carbon feedback) gave 687.7 ppm in 2100 (Enting et al. 1994).

Friedlingstein et al. (2003) expressed the strength of the feedback loop as an amplification factor

$$\kappa = \frac{1}{1 - \frac{\partial T}{\partial C}\frac{\partial C}{\partial T}} = \frac{1}{1-\gamma} \quad (9)$$

where $\frac{\partial T}{\partial C}$ is the temperature response to CO_2 and $\frac{\partial C}{\partial T}$ is the CO_2 response to warming. The product of these responses gives γ, the 'gain' of the loop. Thus compared to the non-feedback situation, a multiplier of κ applies to both the CO_2 response to emissions and the temperature response to other radiative forcing. Actually both components of the feedback loop depend on the time-scales involved. In particular $\frac{\partial T}{\partial C}$ will reflect a transient climate sensitivity. The factor κ applies to $R(t)$ so as to multiply the airborne fraction by a factor of κ. For linear feedbacks, extension of the Laplace transform analysis expresses the strength of the feedback as $\kappa(p)$ to indicate an explicit dependence on the timescale, so that the airborne fraction is increased from $p\ r(p)$ to $p\ \kappa(p)\ r(p)$.

Most of the model calculations reported for the C^4MIP intercomparison (Friedlingstein et al. 2006, Table 2) gave κ values in the range 1.1 to 1.2 with a few cases a little above or below this range, and one model with $\kappa = 1.45$. The higher the apparent κ value, the more likely it becomes that a linear analysis will be inadequate.

To the extent that the uncertainties in the terrestrial budget are being used as a proxy for feedback effects in some modelling, an effective value β_{eff} is being defined with

$$r(p|\beta_{eff}) \approx \kappa(p) r(p|\beta) \quad (10)$$

These considerations become important for the consistent use of carbon cycle data in model calibration. Process-based modelling, calibrated using data such as ^{14}C distributions, will generally estimate the direct behaviour of carbon cycle and not include feedbacks, thus giving an estimate of $R(t)$ or equivalently $p\ r(p)$, the airborne fraction that would apply in the absence of feedbacks. In contrast, direct calibration against atmospheric trends will effectively be estimating $p\ \kappa(p)\ r(p)$, the airborne fraction that reflects the combined effects of twentieth century emissions and warming. As with the example of ocean carbon flux, it is important to ensure that composite data sets are not reflecting inconsistent aspects of the carbon system.

An example of the type of complication that can arise from inconsistent use of data is the calculation of twentieth century CO_2 emissions, $S(t)$, from observed concentrations $C(t)$, using a model with response $R(t)$ calibrated to twentieth century changes without explicit consideration of carbon-to-climate feedbacks. For the timescales captured by the calibration, the estimated sources $\hat{S}(t)$ should represent net anthropogenic forcing, since the feedback is being treated as 'internal' to the model, while on other timescales estimated sources will include a contribution from feedbacks. The size of such contributions to flux estimates will depend on the extent to which approximation (10) fails on a particular timescale of interest.

Concluding remarks

To summarise, consistency in combining information about different aspects of global change is of the utmost importance, since many disparate disciplines are involved in the analysis of human influences and impacts. In particular there is a need for clear definitions, expressed in an agreed common language, of how the boundaries between disciplinary components are treated.

While the examples above have concentrated on the globally aggregated carbon cycle, regional carbon cycle budgeting entails similar consistency requirements. Detailed spatially resolved modelling of the carbon cycle is needed for analysing various aspects of global change, including resolving the uncertainties in climate-to-carbon feedback. A number of classes of carbon cycle data were identified by Canadell et al. (2000) as having the potential to contribute to spatially resolved carbon cycle modelling. In addition, there is the prospect of various sets of concentration data from satellites giving global coverage. In considering the use of such 'multiple constraints' for calibrating models, it is important to recognise the diversity of statistical aspects described by Raupach et al. (2005). The various forms of information need to be combined in ways that are consistent with their respective uncertainties. The data usage also has to be consistent with the form of the modelling that is being undertaken. In terms of the modelling spectrum, spatially resolved models may give the appearance of being at the 'white-box' end of the spectrum, due to the large amount of detail that is captured. However, in reality they may be better considered as multiple realisations of lumped (black-box) models.

Going beyond the behaviour of the carbon cycle, many of the issues discussed above apply much more broadly in the analysis of global change. The issue of consistency in the modelling of past versus future is a generic problem. In particular, the difficulties in ensuring that uncertainties in consequences of future scenarios are consistent with uncertainties concerning the past go beyond the specific case discussed in connection with Eq. 4. In particular, the forcing/response correlation associated with land-use change and CO_2-enhanced growth has a parallel in the correlation between estimated climate sensitivity and twentieth century aerosol forcing, as noted by Allen et al. (2000).

As a pre-requisite for a risk-based analysis, 'consistency' in representing global change needs to include not only the definitions of quantities but also the specifications of ranges and/or probabilities.

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